SYNOPSIS

Some Studies on Fractional Order Random Integro Differential Equations

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Introduction:

Differential and integral equations of fractional order play a very important role in describing some real world problems. For example some problems in physics, mechanics and other fields can be described with the help of differential and integralequations of fractional order (cf. [1,12,17,21,23,25,26,29]).The theory of differential and integral equations of fractional order has recently received a lot of attention and now constitutes a significant branch of nonlinear analysis. Numerous research papers and monographs have appeared devoted to differential and integral equations of fractional order(cf. [1,2,16,18,20,21,23]).

Usually the mathematical models or the equations that have been used to describe a particular phenomenon or process of the universe contains a certain parameter, which has some specific physical interpretation, but whose value is unknown if any such phenomena involving so mentioned parameter and which satisfy certain probabilistic or measure theoretic behavior with respect to the parameter, then we say it is random phenomena. The coefficient or parameters that have an important role in the natural processes are called random parameters. Hence when we talk about some parameters or coefficients, the random analysis of the random equations is evident. Therefore the random equations have been studied in the literature, some long time by various mathematicians all over the world. Thus, the study of a natural or physical phenomenon with the help of random models or equations forms an important branch of the nonlinear analysis.

In recent years random fixed point theory is an important area of research due to its application in various fields. The theory of random fixed point theorems was initiated by Prague School of Probabilistic in 1950s. The interest in this subject enhanced after publication of survey papers by Bharucha-Reid [7], Random fixed point theorems applied in probability theory and nonlinear analysis. The random fixed point theory for self-maps and nonself maps has been further

extended for 1-set contractive, non expansive, semi-contractive and completely continuous random maps etc. [7, 14].

Fractional order differential and integral equations play a very important role in many applications of real world problems, such as Abel's integral equation, viscoelasticity, Schrodinger equation, food science, fractional diffusion equation etc. [15]. In the last few decades, random differential models are found to be more adequate than integer as well as fractional order models for some real world problems. In consequence, the subject of random integral and differential equations is gaining much importance. The study on such random differential and integral equations of fractional order so far made in literature are restricted to existence theory for particular fixed point theorems however other aspects such as existence of extremal solutions, locally attractivity& globally attractivity result of the solutions for such system have been discussed less in the literature. Therefore the present research work proposes to undertake the study of some characterization of solutions of different type of random integral and differential equations of fractional order applying different types of random fixed point theorems. It is claimed that the results of this topics are basic and original contribution to the nonlinear random analysis.

1. Review of literature and development in the subject:

(Previous work done in the relevant type)

The study of a natural or physical phenomenon with the help of random models or equations form an important branch of the nonlinear analysis and some details of random equations and their analysis appear in the famous monographs like Bharucha-Reid[7]. Random differential equation has been the subject of rather extensive among the several eminent mathematicians like Ladde and Lakshmikantham[12] and random differential inequalities by Academic Press, New Yark.

The initial value problem of random differential equations have also been discussed as the literature for existence theorems on bounded intervals, however, the study of such random equations has not been made on unbounded intervals of real line for any aspects of the random solutions. Some results along this lines appear in Itoh [30], Bharucha-Reid [6,7] and Dhage [9].Therefore, the nonlinear random differential equations on un-bounded intervals need to pay attention for existence as well as for different characterizations of the random solutions. to discuss the existence and attractivity results for random differential equations on right half R+ of

the real line R. The classical Fixed point theory, in particular, random version of Schauder's fixed point theorem will be employed to prove the main result.

The problems of nonlinear integral equations have been studied in the literature on bounded as well as unbounded internals of real line for di_erent aspects of the solutions. See for example, Banas andDhage [9], Burton and Zhang [32], Burton and Furumochi [32], Dhage[6,7,9], Hu and Yan [34] and the references therein. Similarly, the initial value problem of random integral equations have also been discussed un the literature for existence theorems on bounded intervals, however, the study of such random equations has not been made on unbounded intervals of real line for any aspects of the random solutions. Some results along this lines appear in Itoh [30], Bharucha-Reid [7] and Dhage [9].

Fixed point theory can be classified in to three main streams, viz algebraic, topological and geometrical fixed point theory. This classification is not rigid but based on the nature of major hypothesis involved in the fixed point theorems. If fixed point theorems involve the mixed hypothesis of algebra, topological and geometry, then it is called hybrid fixed point theorem and these hybrid fixed point theorem constitute a new stream of hybrid fixed point theory in the subject of nonlinear analysis. The hybrid fixed point theorems find numerous applications to other areas of mathematics such as differential and integral inclusion and equations. To the best of our knowledge, Hybrid fixed point theorem for single value mapping initiated by Russian mathematician Kransnoselskii's[20] constitutes the four important category of fixed point theory in the subject of nonlinear analysis. The hybrid fixed point theory for single valued mapping is growing with a good place latter in 1988, the author B. C. Dhage [9] proved a similar type of hybrid fixed point theorem in Banach algebras involving the product of two operators and at present therefore several hybrid fixed point theorems for multivalued mapping available in the literature [4,5,11,30].

It is well known that random fixed point theorems are stochastic generalization of classical fixed point theorems that have been described as deterministic results. Random fixed point theorems for random contraction mappings on separable complete metric spaces were first proved by Spacek and Hans [23]. The article by Bharucha Reid [7] in 1976 attracted the attention of several mathematicians and led to the development of their theory. Spacker's and Hans's theorems have been extended to multivalue contraction mapping by Itoh [30]. A random version

of schaduer fixed point theorem on an atomic probability measure space has been provided by Mukherjee, Bharucha-Reid [6,7].

Itoh [30] obtained random fixed point theorems with an application to random differential equations in Banach spaces. Prakash Rao has obtained the random version of the second important topological fixed point theorem of Kranoselskill's on an atomic probability measure space. Dhage [11] gives a random version of a third important topological fixed point theorems. Kumam is a series of papers [25] proved some remarkable results on random fixed point theorems for multivalue non-expansive nonself-operators in the framework of Banach spaces satisfying inwardness conditions.

The study of global attractivity, asymptotic stability of solution of a nonlinear integral equation in 2006 by Hu. J. Yan. In 2007 J. Banas and B. C. Dhage study the global asymptotic stability of solution of functional integral equations. In 2013 B. D. Karande study the uniform global locally attractive solutions for fractional order random integral equation.

2. Objectives of research / proposed hypothesis :

The present study will be a basic new and original contribution to the theory of random integral & differential equations of fraction order the aim of our present study in the four points.

(i) To study some random integral & differential equations of fractional order via different techniques by fixed point method for their different behavior of the solutions.

(ii) To study existence and uniqueness of solution fractional order random equations.

(iii) To study attractivity results for random integral & differential equations of fractional order.

(iv) To discuss some applications of the nonlinear random integral & differential equations of fractional order studied in the present work of some concrete problems of other areas of mathematics.

Random integral & differential equation of integer order was previously studied long before in the literature. The case of random integral & differential equation of fractional order is completely is new to the literature. In the present work it is proposed to discuss the some of the characterizations of the solutions for fractional order random integral & differential

equations.The present research work also proposes to study some known and some new random integral & differential equations of fraction order.

3. Methodology to be adopted :

There are different approaches or methods for dealing with random equations of integer and fraction order.

(i) Approximation Method

(ii) Measurable Selection Method

(iii) Numerical Method

(iv) Random Fixed Point Method

In the first approach of approximation methods for given fixed point value of the random parameters, the deterministic or classical suitable method and then measurability of the classical solution with respect to this random parameter is proved step by step successively which there by gives the random solution.

This approach has been extended in the work of Ladde and Lakshmikantham [12]. The second approach of measurable selection method is very interesting, but it has a limitation then the measurability of this solution with respect to the random parameter is proved by application of the known results from probabilistic functional analysis Bharcha–Reid [7] and references there in.

The random fixed point method is the approach which is the most powerful one which on application directly yields the existence of random solutions. Numerical techniques are also useful to solve fractional order differential and integral equations.Several numerical methods have been given such as variational iterative method [23],homotopy perturbation method Adomain decomposition method , Euler method, Runge-Kuttamethod.Methodology for dealing with the differential equation consists of the following steps. In the first step, convert the given differential equation into the integral equation, then second step convert obtained integral equation into equivalent operator equations. In the third step we apply some suitable random fixed point theorems of different nature such as topological, algebraic, and geometric will be employed as per the need of situations to the operator equations to yield the existence of solution.

4. Importance of the Study / Society Application :

The present study will be new and original contribution to the subject of nonlinear random differential and integral equation of fractional order and the finding of this work will be useful to research working in the abstract as well as applied field of fractional dynamic system and their applications to real world problem. Ordinary and partial fractional differential equations have been the focus of many studies due to their frequent appearance in various applications in fluid mechanics, viscoelasticity, biology, physics and engineering.

In many problems, properties of certain random variable associated with the solution process are of importance for example in the study of certain models of failure in engineering and biological systems it is important to consider the properties of random variable. It is hoped that the present work will be useful to scientists, engineers and mathematicians, who are engaged in analyzing the universal random phenomena govern by nonlinear random problems of integral and differential equations in better way.

5. Proposed Work Plan / Formulation and Structure of Study:

Year wiseplan of work and target to be achieved.

First year Plan: In the first year the researcher will attend some seminar/ conference and complete course work in the first stage of research. The researcher tries to build up the necessary background to understand the area of his research and getting a broad view about research problem. Researcher survey the literature related to research topic. The researcher will collect material for research such as journals, books, papers, different solve and unsolved problems etc.

Second Year Plan: The collected material will be analyzed to discover the finding directions of research. Different methodology tools techniques will be used in this stage for solving the new research problems. After the conclusion the researcher will publish his research findings in national and international journals. The researcher will visit libraries, prominent mathematicians to verify his findings.

Third Year Plan: The outcomes of our research work done will be presented in national and international gathering such as seminars, workshops and conference during this year. The research work will be published in several journals and thesis writing.

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